4-2 Triangle Congruence by SSS and SAS

Mathematics Florida Standards

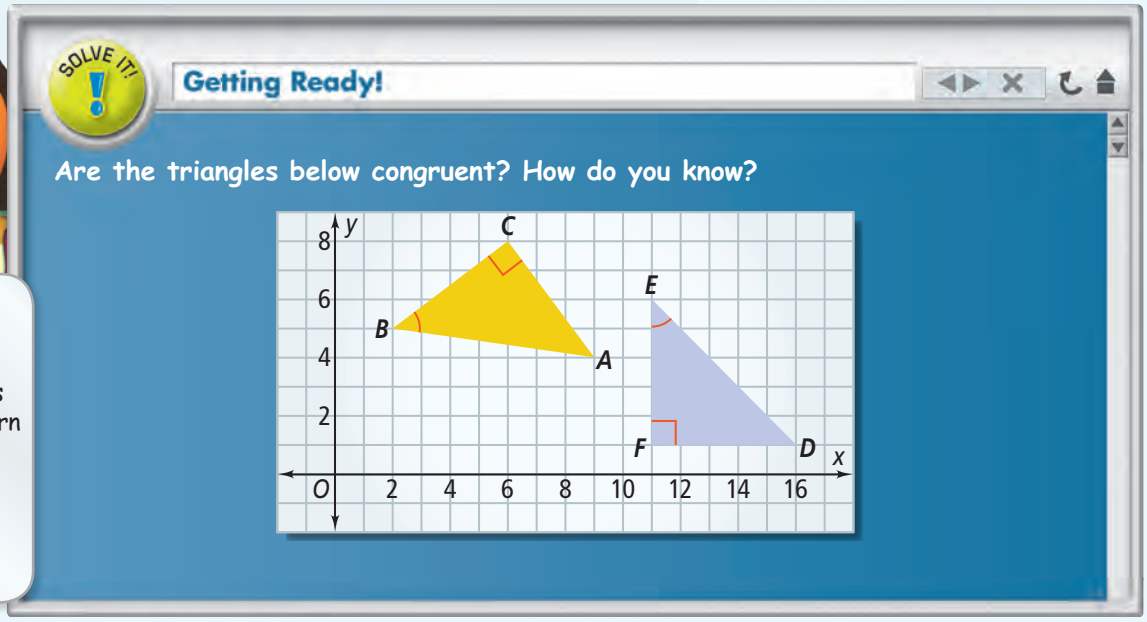
MAFS.912.G-SRT.2.5 Use congruence criteria for triangles to solve problems and prove relationships in geometric figures.

MP 1, MP 3, MP 4, MP 7

Objective To prove two triangles congruent using the SSS and SAS Postulates

How can you tell whether these triangles are congruent? In this lesson, you will learn the least amount of information required to tell if two triangles are congruent.

MATHEMATICAL PRACTICES



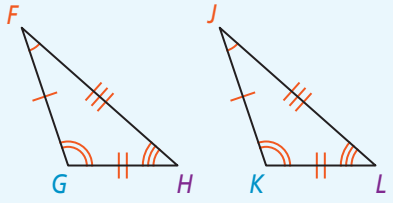
In the Solve It, you looked for relationships between corresponding sides and angles. In Lesson 4-1, you learned that if two triangles have three pairs of congruent corresponding angles and three pairs of congruent corresponding sides, then the triangles are congruent.

If you know . . .

$\angle F \cong \angle J$ $\quad \overline{FG} \cong \overline{JK}$

$\angle G \cong \angle K$ $\quad \overline{GH} \cong \overline{KL}$

$\angle H \cong \angle L$ $\quad \overline{FH} \cong \overline{JL}$



. . . then you know $\triangle FGH \cong \triangle JKL$.

However, this is more information about the corresponding parts than you need to prove triangles congruent.

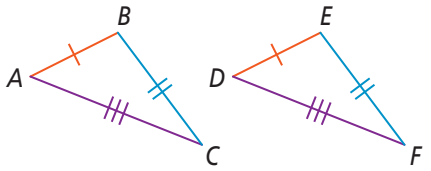
Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove triangles congruent by using (1) three pairs of corresponding sides and (2) two pairs of corresponding sides and one pair of corresponding angles.

take note

Postulate 4-1 Side-Side-Side (SSS) Postulate

Postulate
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

If . . .
$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$



Then . . .
$\triangle ABC \cong \triangle DEF$

As described in Chapter 1, a postulate is an accepted statement of fact. The Side-Side-Side Postulate is perhaps the most logical fact about triangles. It agrees with the notion that triangles are rigid figures; their shape does not change until pressure on their sides forces them to break. This rigidity property is important to architects and engineers when they build things such as bicycle frames and steel bridges.

Proof

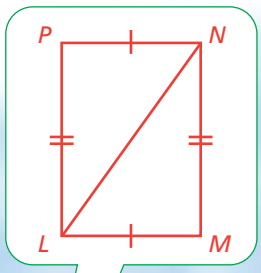
Problem 1 Using SSS

Given: $\overline{LM} \cong \overline{NP}$, $\overline{LP} \cong \overline{NM}$

Prove: $\triangle LMN \cong \triangle NPL$

Plan

You have two pairs of congruent sides. What else do you need?
You need a third pair of congruent corresponding sides. Notice that the triangles share a common side, $\overline{LN}$.



$\overline{LM} \cong \overline{NP}$ Given

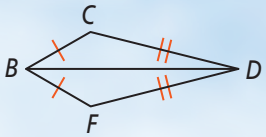
$\overline{LN} \cong \overline{LN}$ Reflexive Prop. of $\cong$

$\overline{LP} \cong \overline{NM}$ Given

$\triangle LMN \cong \triangle NPL$ SSS

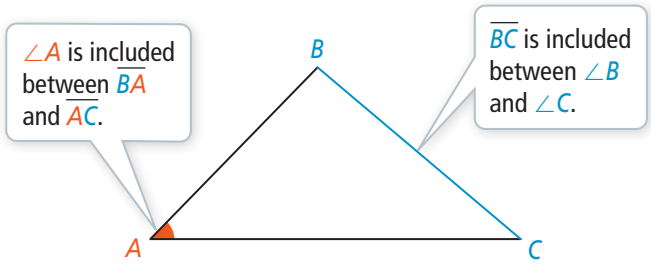
Got It? 1. **Given:** $\overline{BC} \cong \overline{BF}$, $\overline{CD} \cong \overline{FD}$

Prove: $\triangle BCD \cong \triangle BFD$



You can also show relationships between a pair of corresponding sides and an *included* angle.

The word *included* refers to the angles and the sides of a triangle as shown at the right.

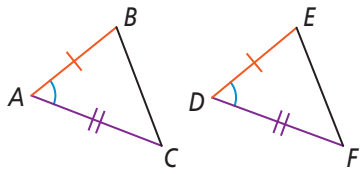


take note

Postulate 4-2 Side-Angle-Side (SAS) Postulate

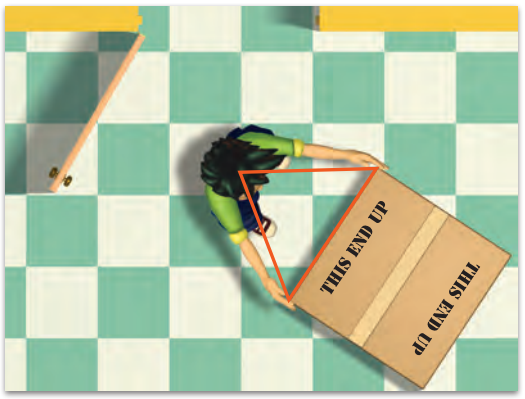
Postulate
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

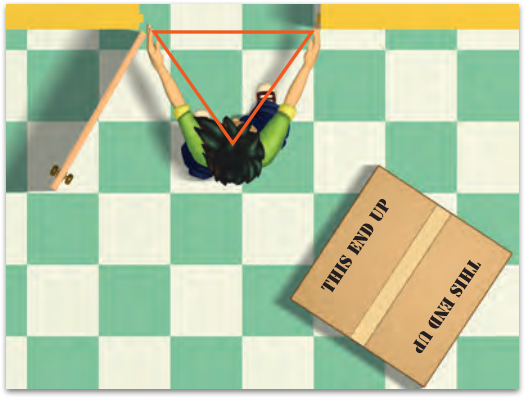
If . . .
$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$



Then . . .
$\triangle ABC \cong \triangle DEF$

You likely have used the properties of the Side-Angle-Side Postulate before. For example, SAS can help you determine whether a box will fit through a doorway.





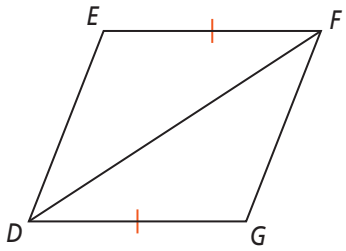
Suppose you keep your arms at a fixed angle as you move from the box to the doorway. The triangle you form with the box is congruent to the triangle you form with the doorway. The two triangles are congruent because two sides and the included angle of one triangle are congruent to the two sides and the included angle of the other triangle.

Problem 2 Using SAS

Plan

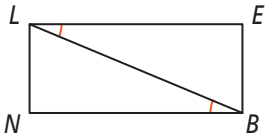
Do you need another pair of congruent sides?
Look at the diagram. The triangles share $\overline{DF}$. So, you already have two pairs of congruent sides.

What other information do you need to prove $\triangle DEF \cong \triangle FGD$ by SAS? Explain.

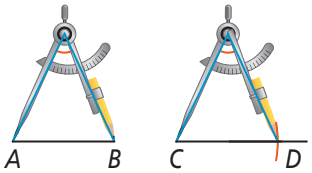


The diagram shows that $\overline{EF} \cong \overline{GD}$. Also, $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence. To prove that $\triangle DEF \cong \triangle FGD$ by SAS, you must have congruent included angles. You need to know that $\angle EFD \cong \angle GDF$.

Got It? **2.** What other information do you need to prove $\triangle LEB \cong \triangle BNL$ by SAS?



Recall that, in Lesson 1-6, you learned to construct segments using a compass open to a fixed angle. Now you can show that it works. Similar to the situation with the box and the doorway, the Side-Angle-Side Postulate tells you that the triangles outlined at the right are congruent. So, $\overline{AB} \cong \overline{CD}$.



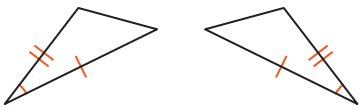
Problem 3 Identifying Congruent Triangles

Plan

What should you look for first, sides or angles?
Start with sides. If you have three pairs of congruent sides, use SSS. If you have two pairs of congruent sides, look for a pair of congruent included angles.

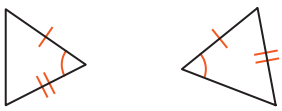
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*. Explain your answer.

A



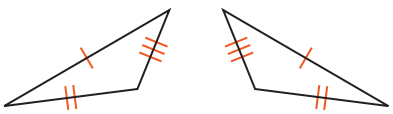
Use SAS because two pairs of corresponding sides and their included angles are congruent.

B



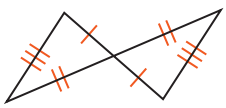
There is not enough information; two pairs of corresponding sides are congruent, but one of the angles is not the included angle.

C



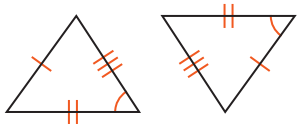
Use SSS because three pairs of corresponding sides are congruent.

D



Use SSS or SAS because all three pairs of corresponding sides and a pair of included angles (the vertical angles) are congruent.

Got It? **3.** Would you use SSS or SAS to prove the triangles at the right congruent? Explain.

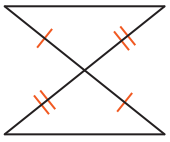


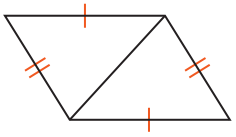
Lesson Check

Do you know HOW?

1. In $\triangle PEN$, name the angle that is included between the given sides.
   a. $\overline{PE}$ and $\overline{EN}$ b. $\overline{NP}$ and $\overline{PE}$
2. In $\triangle HAT$, between which sides is the given angle included?
   a. $\angle H$ b. $\angle T$

Name the postulate you would use to prove the triangles congruent.

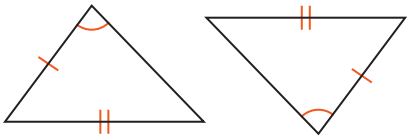
3. 

4. 

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Compare and Contrast** How are the SSS Postulate and the SAS Postulate alike? How are they different?

6. **Error Analysis** Your friend thinks that the triangles shown below are congruent by SAS. Is your friend correct? Explain.

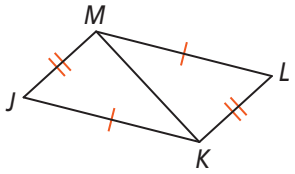


7. **Reasoning** A carpenter trims a triangular peak of a house with three 7-ft pieces of molding. The carpenter uses 21 ft of molding to trim a second triangular peak. Are the two triangles formed congruent? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

8. **Developing Proof** Copy and complete the flow proof.



Given: $\overline{JK} \cong \overline{LM}$, $\overline{JM} \cong \overline{LK}$

Prove: $\triangle JKM \cong \triangle LMK$

$\overline{JK} \cong \overline{LM}$ — Given

$\overline{JM} \cong \overline{LK}$ — a. ?

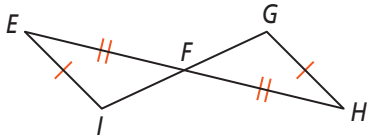
$\overline{KM} \cong \overline{KM}$ — b. ?

c. ? $\cong$ d. ? — SSS

See Problem 1.

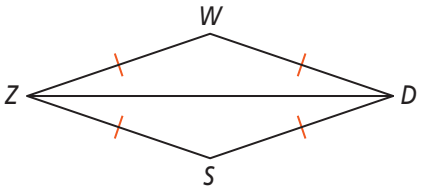
9. Proof **Given:** $\overline{IE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HF}$, F is the midpoint of $\overline{GI}$

Prove: $\triangle EFI \cong \triangle HFG$



10. Proof **Given:** $\overline{WZ} \cong \overline{ZS} \cong \overline{SD} \cong \overline{DW}$

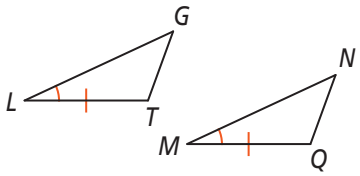
Prove: $\triangle WZD \cong \triangle SDZ$



What other information, if any, do you need to prove the two triangles congruent by SAS? Explain.

See Problem 2.

11.



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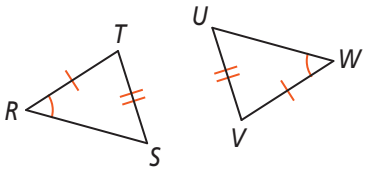
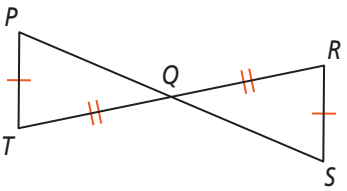


Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*. Explain your answer.

See Problem 3.

13.



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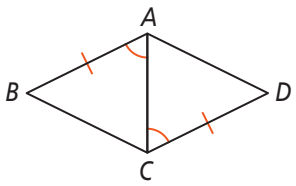
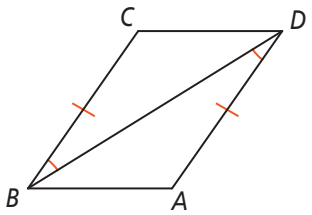


B Apply

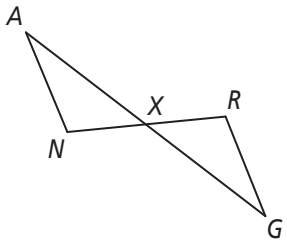
15. Think About a Plan You and a friend are cutting triangles out of felt for an art project. You want all the triangles to be congruent. Your friend tells you that each triangle should have two 5-in. sides and a 40° angle. If you follow this rule, will all your felt triangles be congruent? Explain.

- How can you use diagrams to help you?
- Which postulate, SSS or SAS, are you likely to apply to the given situation?

16. Proof **Given:** $\overline{BC} \cong \overline{DA}$, $\angle CBD \cong \angle ADB$
Prove: $\triangle BCD \cong \triangle DAB$



17. Proof **Given:** X is the midpoint of $\overline{AG}$ and $\overline{NR}$.
Prove: $\triangle ANX \cong \triangle GRX$



Use the Distance Formula to determine whether $\triangle ABC$ and $\triangle DEF$ are congruent. Justify your answer.

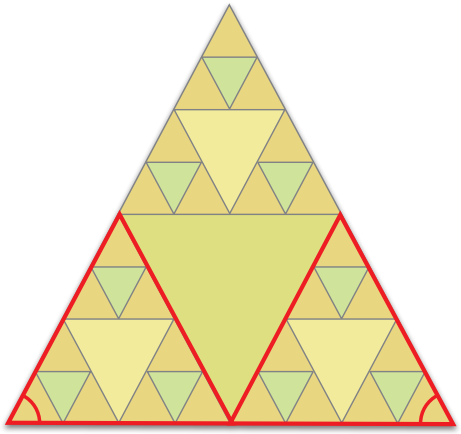
18. $A(1, 4)$, $B(5, 5)$, $C(2, 2)$; $D(-5, 1)$, $E(-1, 0)$, $F(-4, 3)$

19. $A(3, 8)$, $B(8, 12)$, $C(10, 5)$; $D(3, -1)$, $E(7, -7)$, $F(12, -2)$

20. $A(2, 9)$, $B(2, 4)$, $C(5, 4)$; $D(1, -3)$, $E(1, 2)$, $F(-2, 2)$

21. Writing List three real-life uses of congruent triangles. For each real-life use, describe why you think congruence is necessary.

22. Sierpinski's Triangle Sierpinski's triangle is a famous geometric pattern. To draw Sierpinski's triangle, start with a single triangle and connect the midpoints of the sides to draw a smaller triangle. If you repeat this pattern over and over, you will form a figure like the one shown. This particular figure started with an isosceles triangle. Are the triangles outlined in red congruent? Explain.



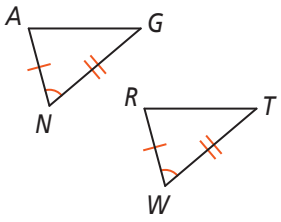
23. Constructions Use a straightedge to draw any triangle JKL. Then construct $\triangle MNP \cong \triangle JKL$ using the given postulate.

a. SSS

b. SAS

Can you prove the triangles congruent? If so, write the congruence statement and name the postulate you would use. If not, write *not enough information* and tell what other information you would need.

24.



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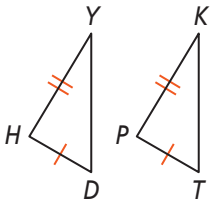

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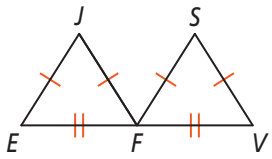
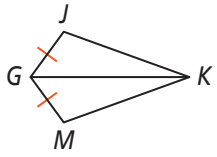


27. Reasoning Suppose $\overline{GH} \cong \overline{JK}$, $\overline{HI} \cong \overline{KL}$, and $\angle I \cong \angle L$. Is $\triangle GHI$ congruent to $\triangle JKL$? Explain.

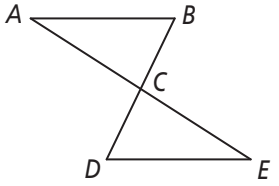
28. Proof **Given:** $\overline{GK}$ bisects $\angle JGM$, $\overline{GJ} \cong \overline{GM}$

Prove: $\triangle GJK \cong \triangle GMK$



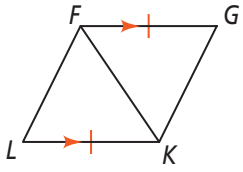
29. Proof **Given:** $\overline{AE}$ and $\overline{BD}$ bisect each other.

Prove: $\triangle ACB \cong \triangle ECD$



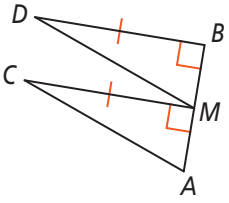
30. Proof **Given:** $\overline{FG} \parallel \overline{KL}$, $\overline{FG} \cong \overline{KL}$

Prove: $\triangle FGK \cong \triangle KLF$



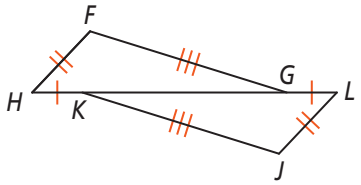
31. Proof **Given:** $\overline{AB} \perp \overline{CM}$, $\overline{AB} \perp \overline{DB}$, $\overline{CM} \cong \overline{DB}$, M is the midpoint of $\overline{AB}$

Prove: $\triangle AMC \cong \triangle MBD$

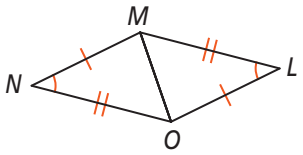


Challenge

32. **Proof** **Given:** $\overline{HK} \cong \overline{LG}$, $\overline{HF} \cong \overline{LJ}$, $\overline{FG} \cong \overline{JK}$
Prove: $\triangle FGH \cong \triangle JKL$



33. **Proof** **Given:** $\angle N \cong \angle L$, $\overline{MN} \cong \overline{OL}$, $\overline{NO} \cong \overline{LM}$
Prove: $\overline{MN} \parallel \overline{OL}$

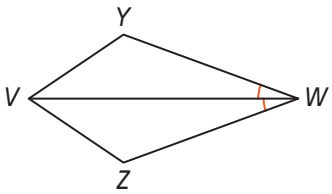


34. **Reasoning** Four sides of polygon *ABCD* are congruent, respectively, to the four sides of polygon *EFGH*. Are *ABCD* and *EFGH* congruent? Is a quadrilateral a rigid figure? If not, what could you add to make it a rigid figure? Explain.

Standardized Test Prep

SAT/ACT

35. What additional information do you need to prove that $\triangle VWY \cong \triangle VWZ$ by SAS?



(A) $\overline{YW} \cong \overline{ZW}$
(B) $\angle WVY \cong \angle WVZ$
(C) $\angle Y \cong \angle Z$
(D) $\overline{VZ} \cong \overline{VY}$

36. The measures of two angles of a triangle are 43 and 38. What is the measure of the third angle?

(F) 9 (G) 81 (H) 99 (I) 100

37. Which method would you use to find the inverse of a conditional statement?

(A) Switch the hypothesis and conclusion.
(B) Negate the hypothesis only.
(C) Negate the conclusion only.
(D) Negate both the hypothesis and conclusion.

Short Response

38. A segment has a midpoint at (1, 1) and an endpoint at (−3, 4). What are the coordinates of the other endpoint of the segment? Show your work.

Mixed Review

ABCD ≅ *EFGH*. **Name the angle or side that corresponds to each part.** See Lesson 4-1.

39. $\angle A$ **40.** $\overline{EF}$ **41.** $\overline{BC}$ **42.** $\angle G$

Write the converse of each statement. Determine whether the statement and its converse are true or false. See Lesson 2-2.

43. If $x = 3$, then $2x = 6$.

44. If $x = 3$, then $x^2 = 9$.

Get Ready! **To prepare for Lesson 4-3, do Exercises 45 and 46.**

45. In $\triangle JHK$, name the side that is included between $\angle J$ and $\angle H$. See Lesson 4-2.

46. In $\triangle NLM$, name the angle that is included between $\overline{NM}$ and $\overline{LN}$.